AP Statistics
Test: Introduction to Inference
Instructor's Version

Free Response: 2 questions, 52 points, 30 minutes

1. A tire manufacturer wishes to test a sample of its best-selling tire for average lifetime mileage. When the manufacturing process is operating properly, the standard deviation of the mileage is known to be $\sigma = 2{,}500$. An SRS of 100 tires is selected and is found to have mean mileage $\bar{x} = 59{,}500$. You wish to construct a 95% confidence interval for μ.

A. Identify μ, the parameter about which we want to draw conclusions. (6 points)

Answer:
μ is the mean lifetime mileage of all the best-selling tires that the tire manufacturer produces.

B. Verify that the necessary conditions exist for construction of a confidence interval. (4 points)

Answer:
1. SRS was stated in the problem
2. Large sample size, *n* = 100 (generally 30 or greater is considered large)

Scoring notes: Award 2 points for each condition stated and verified.

C. Calculate a 95% confidence interval for μ. (6 points)

Answer:
95% confidence interval is (59010, 59990) and can be calculated with calculator or by hand using the formula for confidence interval:
$\bar{x} \pm z \bullet \frac{\sigma}{\sqrt{n}} = 59{,}500 \pm 1.96\left(\frac{2{,}500}{\sqrt{100}}\right) =$ (59010, 59990).

Scoring notes: Award 6 points for a correct confidence interval with work shown. Award up to 1 point only if no work shown.

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D. Interpret the confidence level of 95%. (6 points)

Answer:
Note: There are several possible correct answers.

95% of all possible random samples of size 100 from this population will result in a confidence interval that includes the true population mean mileage.
OR
It can be estimated with 95% confidence that the true population mean lifetime mileage of the tires is between 59,010 and 59,990.
OR
The method used to produce this interval will fail to capture the true population mean about 5% of the time.
OR
Approximately 95% of a large number of intervals will contain the true population mean.

E. What will happen to your confidence interval if your confidence level is raised to 99%? (2 points)

Answer:
The confidence interval will be wider.

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2. Using the tire manufacturer from problem No. 1, the manufacturer claims that the mean lifetime mileage for this particular brand of tires is 60,000 miles. A consumer group believes the mean lifetime mileage is less than 60,000 miles.

A. Is there sufficient evidence to support the manufacturer's claim of a mean of 60,000 miles? Carry out an appropriate test at a 5% significance level. (20 points)

NOTE: It is not necessary to verify conditions.

Answer:
Hypotheses are: $H_o : \mu = 60{,}000$ and $H_\alpha : \mu = 60{,}000$

Calculation of test statistic: $z = \dfrac{\overline{x - \mu}}{\dfrac{\sigma}{\sqrt{n}}} = \dfrac{59{,}500 - 60{,}000}{\dfrac{2500}{\sqrt{100}}} = \dfrac{-500}{250} = -2$

p-value (from calculator or table) = 0.0228
normalcdf(-E99,-2) = 0.0228

Decision: Reject the null hypothesis because *p*-value, 0.0228 0.05, sig.level.

Conclusion: No, there is not sufficient evidence to support the manufacturer's claim of mean mileage of 60,000.

Scoring notes:
Each of the five parts is worth a maximum of 4 points.

B. Would your conclusion be any different if the test were carried out at a 1% significance level? Explain. (4 points)

Answer:
Yes, the answer would be different. At a 1% significance level, we would fail to reject the null hypothesis since *p*-value, 0.0228, is not less than 0.01. We would then have evidence to support the manufacturer's claim.

C. Explain, in words, what would constitute a type I error in the context of the problem. (4 points)

Answer:
Since a type I error occurs when a correct null hypothesis is rejected, we would state that the mean lifetime mileage for the tires is not 60,000 miles (when it actually is).

Scoring notes:
Context must be given for full credit. Award 2 points only for definition of type I error without context.

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